UNIT 5

LINEAR EQUATIONS, LINEAR INEQUALITIES AND PROPORTIONALITY

Unit outcomes: **After completing this unit you should be able to:**

- develop your skills in solving linear equations and inequalities (of the form $x+a=b$, $x+a>b$).
- understand the concept of direct and inverse proportionalities and represent them graphically.

Introduction

In grade 4 mathematics lessons, you have learnt about a mathematical expression. In the present unit, you will learn about solution of simple linear equations and inequalities. You shall also learn about the concept of direct and inverse proportionalities and representing them graphically.

5.1 Solution of Simple Linear Equations and Inequalities

5.1.1 Solution of One-step Linear Equations

Activity 5.1

1. Write a mathematical expression for each phrase.

   i) 6 plus k __________
   vi) 54 divided by d ___________
   ii) x more than 18__________
   v) 16 times h___________
   iii) Y less than 10 _____

2. Evaluate each expression, if a=6, b=3, and c=2.

i) a+b+c | v) $\frac{6(a+c)}{b}$

ii) 4ab-c | vi) c(b+a)-a

iii) 2(a+b)-c

3. Match the terms in column A with its simplified expression in column B

Column A	Column B
i. $2x + 7 + 5x - 4 - x$	a) $5y + 9x + 12$
ii. $5 + 7x + 2x - 3 + 6$	b) $12y + 6x + 24$
iii. $x + y + 4x - 3x + 2y + 3y$	c) $9x+8$
iv. $3x^2 + 5x - 17 + 6x + 20$	d) $3x^2$
v. $4x + x^2 + 12 - 4 + 2x$	e) $6x + 3$
vi. $12y + 12x + 12 - 6x + 12$	f) $3x^2 + 11x + 3$
vii. $12y + 4 + x - 7y + 8 + 8x$	g) $x^2 + 6x + 8$
viii. $5x + x^2 + 2x + 4 - 4 - x^2$	h) $7x$
ix. $5x^2 + 8x + 7x^2 + 6 - 12x^2$	i) $2x + 6y$
x. $x^2 + 3 + 2x^2 + 4 - 7$	j) $8x+6$
	k) $9x + 18$
	l) $3x + 6$

4. Identify each of the following as equation or inequality.

a) $x + 1 = 3$ | c) $5a = 10$

b) $2x > 5$ | d) $x-1 < 4$

Can you tell the difference between a mathematical expression, equation and inequality? You have learnt about a mathematical expression, an equation and an inequality in grade 4 mathematics lessons. Here you will study about solution of one-step linear equations.

Definition 5.1: An equation that can be written in the form $ax+b=0$, $a \neq 0$ is called a **linear equation**.

Example 1

Equations such as 2x+3=0, 3x-5=0, 10x=10 and x+7=0 are linear equations in one variable.

In the linear equation x+2=6, for example, the variable x represents the number or unknown for which we are solving.

We **solve** the equation when we replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution or a root of the equation**. The solution to x+2=6 is 4 because 4+2= 6 is true.

Example 2

Which of the numbers 8,9 or 10 is the solution of 9+x=19?

Solution:

Replace x with 8	Replace x with 9	Replace x with 10
9+x=19	9+x=19	9+x=19
9+8=19	9+9=19	9+10=19
$17 \neq 19$	$18 \neq 19$	19=19
This sentence is false.	This sentence is false.	The sentence is true.

Therefore, the solution is 10.

Definition 5.2: The set whose elements are considered as possible replacement for the variable in a given equation or inequality is called the **domain of the variable**.

For example, in Example 2 above, the domain of the variable ={8,9,10}. Usually in solving for the unknown, we place variable(s) on the left side of the equation and constants on the right.

Activity 5.2.

By trial –and –error, find a value of the variable.

i) x+3=8	ii) y+6=0	iii) z-5=0
iv) 7=x-1	v) 5a=20	vi) 23-n=20

The trial-and-error method can be used only for simple equations. For large value of x it is not suitable. Even for simple equation it is time-consuming and tedius too. So, it is better to find a simple and systematic method of solving an equation.

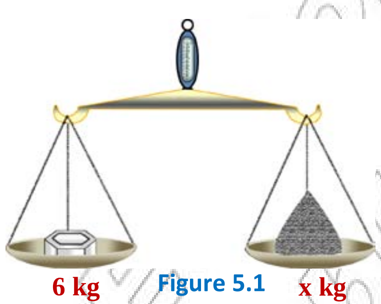


Figure 5.1

To visualize how equations work, think of the balancing scale shown in Figure.5.1. We know that when equal weights are placed on two pans, the beam of the balance remains horizontal. Similarly, two sides of an equation are equal when we have equal numbers on both sides. Suppose, we have x kg of sugar on the right hand pan and to keep it in a balance position we put 6 kg of weight to the left. Then we can say that x=6.

I) Consider Figure 5.2

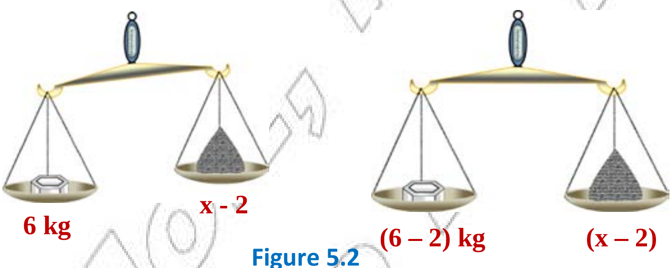


Figure 5.2

If we take away 2kg of sugar, the balance would dip to the left. Clearly, if we remove 2kg of weight from the left pan then only the balance will return to the horizontal position.

In equation form, we can write.

$$x = 6$$
$$x-2 = 6-2 \text{ (Taking away 2 from both sides)}$$
$$\text{or } x-2=4$$

II) Consider Figure 5.3

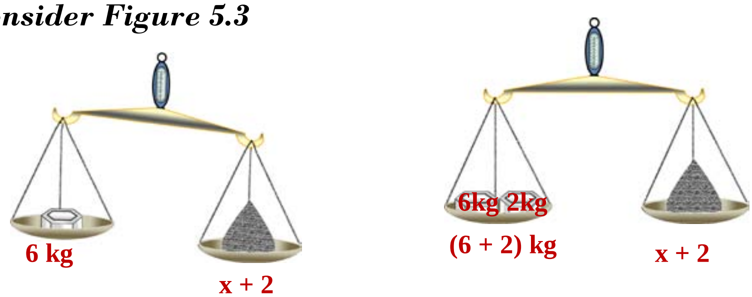


Figure 5.3

If we add 2 kg more sugar to the right pan, the balance would dip to the right. To bring it back to the horizontal position, we have to put 2 kg of weight on the left pan.

In equation form, we can write this as:

$$x = 6$$
$$x+2 = 6+2$$
$$\text{or } x+2 = 8$$

III) Consider Figure. 5.4

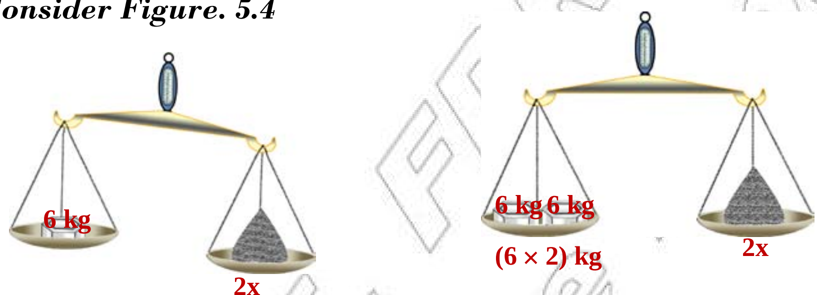


Figure 5.4

If we double the quantity of sugar, the balance would dip to the right pan due to heavy mass and clearly we have to put double weight in the left pan to keep the balance in horizontal position.

In equation form, we can write this as:

$$x = 6$$
$$\text{Or } 2x = 6 \times 2$$
$$\text{Or } 2x = 12$$

IV) Consider Figure 5.5

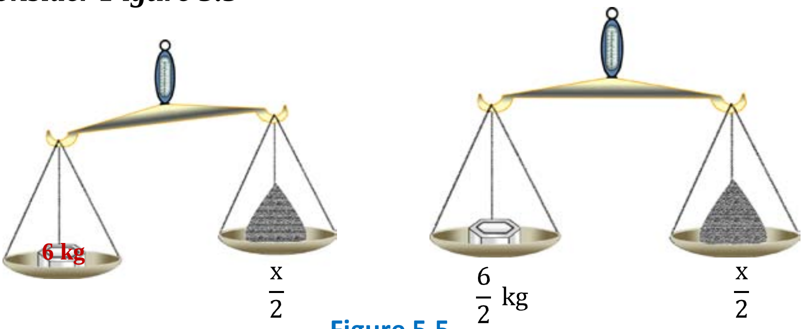


Figure 5.5

If we take away half of the sugar, the quantity of sugar in the right pan becomes less. As a result, the balance will dip to the left. To bring the balance in horizontal position, we have to take half of the weight from the left pan. Therefore, we write $\frac{x}{2} = \frac{6}{2}$ or $\frac{x}{2} = 3$.

In all the above four cases, we make the balance in horizontal position by adjusting the weight with corresponding quantity of sugar. Same process can be applied to an equation, too, under the following **rules**.

1. If we add or subtract the same number to or from both sides of an equation, then the equality symbol will not change.

   For example, a) If x-2=3, then x-2+2=3+2.

   Or x=5

   b) If x+2=3, then x+2-2=3-2 or x=1.

Since multiplication is repeated addition and division is repeated subtraction, we can also apply the following rules:

2. The equality symbol will not be changed in the equation if we multiply both sides of the equation by the same non-zero number.

   For example, if $\frac{x}{2} = 3$, then $\frac{x}{2} \times 2 = 3 \times 2$ or x=6.

3. The equality symbol will not be changed in the equation, if we divide both sides of the equation by the same (non-zero) number.

   For example, if 4x=8, then

   $\frac{4x}{4} = \frac{8}{4}$ or $x = 2$.

The following examples will illustrate how the rules are used in solving the given equations.

Example 3

Solve a) $x+5=13$ c) $3x=24$

b) $x-7=19$ d) $\frac{x}{8}=5$

Solution:

a) $x+5=13$ ....Given

$x+5-5=13-5$ ...... Subtracting 5 from both sides of the equation

Or $x=8$

Check: $8+5=13$(True)

Therefore, $x=8$ is the root or solution of the given equation.

b) $x-7=19$..............Given

$x-7+7=19+7$........Adding 7 to both sides of the equation.

$x=26$

Check: $26-7=19$ (True)

Therefore, $x=26$ is the root or solution of the given equation.

c) $3x=24$................. Given

$\frac{3x}{3}=\frac{24}{3}$ .... Dividing both sides of the equation by 3 or $x=8$

Check: $3\times 8=24$ (True)

Therefore, $x=8$ is the root or solution of the given equation.

d) $\frac{x}{8} = 5$

$8\left(\frac{x}{8}\right) = 8(5)$................Multiplying both sides of the equation by 8

Or x=40

Check: $\frac{40}{8} = 5(\text{True})$

Therefore, x=40 is the root or solution of the given equation.

Example 4

Write and solve the equation

a) A number less three is 14. Find the number.
b) The product of a number and 6 is 84. Find the the number.

Solution

a) Let x represent the number. Then, the equation can be given by x-3=14.

x-3+3=14+3...... Adding 3 to both sides of the equation.

Or x=17

Check: 17-3=14(True)

Therefore, x=17 is the solution.

b) Let x represent the number. Then, the equation can be given by

6x=84

$\frac{6x}{6} = \frac{84}{6}$...... Dividing both sides of the equation by 6

Or x=14

Check: 6(14)=84(True)

Therefore, x=14 is the solution.

Exercise 5 A

1. Solve

a) x-3=9

b) x-7=14

c) x+9=37

d) 16-y=5

e) 34-x=0

f) 2= y-17

g) $\frac{y}{17} = 4$

h) 13-m=59

i) $\frac{10}{3}x = 20$

j) $\frac{2}{5}y = 4$

k) $\frac{11}{6}m = 3$

l) $\frac{4}{7}n = 8$

m) $\frac{n}{10} = 4$

n) 200 x=0.1

o) 0.01n=10

2. Write and solve an equation.

a) 7 more than a number is 34.

b) 3 less than a number is 19.

c) Twice a number is 26.

d) 5 subtracted from 8 times a number gives 0.

e) The number of grade six students at a school is 316. This is 27 more than the number of grade eight students. How many grade eight students are enrolled?

f) An object on the moon weighs one sixth as much as it does on earth. If a person weighs 12 kg on the moon, how much does he weigh on earth?

g) A designer dress costs Birr 225. This is three times the cost of another dress. What is the cost of the less expensive dress?

5.1.2 Solution of One-step Linear Inequalities

Activity 5.3

Write an inequality for each situation.

a) There are at least 20 people in the waiting room.

b) No more than 150 people can occupy the room.

Do you remember what you have studied about inequality in your previous mathematics lessons? You have learnt that mathematical expressions which contain the relation symbols $<$, $\leq$, $>$, $\geq$ or $\neq$ are called **inequalities**. Here you will learn more about inequalities and find out rules for solving linear inequalities.

Definition 5.3: A linear Inequality is inequality of the form $ax + b > 0$, or $ax + b \geq 0$ or $ax + b < 0$, or $ax + b \leq 0$, where $a \neq 0$.

Example 5

Inequalities such as $x+3>0$, $x-4\leq0$, $2x-1<0$ and $3x+7\geq0$ are linear inequalities.

Before we discuss the rules of transformation of inequalities, let us solve a linear inequality by substituting values from a given list of numbers.

Example 6

Which of the following numbers satisfy the inequality $x+3<5$?

a) -4 b) -1 c) 0 d) 2 e)3

Solution

a) $x= -4$, $-4+3<5$
$-1<5$ (True)

b) $x=-1$, $-1+3<5$
$2<5$(True)

c) $x=0$, $0+3<5$
$3<5$ (True)

d) $x=2$, $2+3<5$
$5<5$ (False)

e) $x=3$, $3+3<5$
$6<5$(False)

Thus, -4, -1 and 0 satisfy the given inequality.

Group work 5.1

Study each of the following pairs of inequalities given below carefully.

Left	Right
a) $5+3<9$	$(5+3)+\frac{1}{2} < 9 + \frac{1}{2}$
b) $\frac{2}{3} < \frac{5}{3}$	$\frac{2}{3} - \frac{1}{3} < \frac{5}{3} - \frac{1}{3}$
c) $4 > 2$	$4 \times 3 > 2 \times 3$
d) $10 < 15$	$10 \div 5 < 15 \div 5$

What do you observe? Is the one on the left true? Is the one on the right true?

What was done to the one on the left to obtain the one on the right?

If you have observed carefully, you see the following **rules of transformation**.

1. **Adding or subtracting the same number to or from each side of an inequality keeps the inequality sign remain as it is.**
2. **Multiplying or dividing both sides of an inequality by the same positive number keeps the inequality sign as it is.**

Let us use the above rules of transformation in order to solve inequalities.

Example 7

Solve

$x+4<7$ if the domain is

i) The set of whole numbers

ii) The set of counting numbers

Solution: $x+4<7$

i) $x+4-4<7-4$ ……. Subtracting 4 from both sides of the inequality

Or x<3

i) **The solution of the inequality x+4<7, when the domain is the set of whole numbers, consists of all whole numbers less than 3. That is, 0,1 and 2 are the solutions of the given inequality. Thus, solution set=** $\{0, 1, 2\}$.

ii) **The solution of the inequality x+4<7, when the domain is the set of counting numbers, consists of all counting numbers less than 3. Thus, 1 and 2 are the solutions of the given inequality. That is, solution set=**$\{1, 2\}$.

Example 8

Solve

a) **x-2>5 on the set of whole numbers.**

b) **2x<10 on the set of natural numbers.**

c) $\frac{1}{4}x > 3$ **if** $x \in \{0, 1, 2 \dots, 20\}$.

d) $x + \frac{7}{8} < 1$ **on the set of natural numbers.**

Solution: a) x-2>5

x-2+2>5+2..... Adding 2 to both sides of the inequality

or x>7 and x ∈ W

Thus, solution set = $\{8, 9, 10, \dots\}$.

b) 2x<10

$\frac{2x}{2} < \frac{10}{2}$ ... Dividing both sides of the inequality by 2.

Or x <5 and x∈N

Thus, solution set = $\{1, 2, 3, 4\}$.

c) $\frac{1}{4}x > 3$

$4\left(\frac{1}{4}x\right) > 4(3)$ … **Multiplying both sides of the inequality by 4**

Or x>12 and x$\in\{0, 1, 2, \dots, 20\}$

Thus, solution set =$\{13, 14, 15, 16, 17, 18, 19, 20\}$

d) x+$\frac{7}{8} < 1$

x+$\frac{7}{8} - \frac{7}{8} < 1 - \frac{7}{8}$…. **Subtracting** $\frac{7}{8}$ **from both sides of the inequality**

x< $\frac{1}{8}$ **and x is a counting number**

But there is no counting number which is less than $\frac{1}{8}$.

Therefore, solution set = {}.

Note: The above example illustrates that the existence of the solution set of an inequality depends on the domain of the variable.

Remember also that the solutions of an inequality are the values that make the inequality true. They can be indicated on a number line.

Study the following examples.

Example 9

Solve and indicate the solution on the number line.

a) x- $\frac{3}{4} < \frac{9}{4}$, **domain = The set of whole numbers**

b) x+2<0, domain =The set of integers

Solution: a) x - $\frac{3}{4} < \frac{9}{4}$

x< $\frac{9}{4} + \frac{3}{4}$ **Why?**

x<3, x is a whole number

Therefore, solution set= $\{0, 1, 2\}$

Its graph can be seen from Figure 5.6 below.

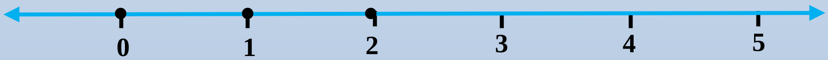


Figure 5.6

b) x+2<0

x+2-2<0-2 (why?)

Or x<-2, x is an integer

Therefore, solution set= $\{\ldots, -5, -4, -3\}$.

Its graph can be seen from figure 5.7 below

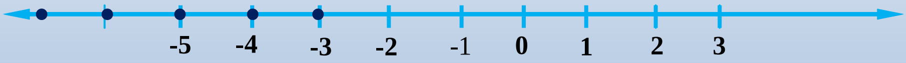


Figure 5.7

Exercise 5-B

1. Solve each of the following inequalities on the given domain.
   a) x+4 < 8, Domain = The set of whole numbers.
   b) y-2 < 7, Domain = The set of integers.
   c) y-3 >-5, Domain = The set of natural numbers.
   d) $\frac{1}{5}$y > 2, Domain = The set of negative integers.
   e) $\frac{2}{3}a < 4$, Domain = The set of whole numbers.
   f) $\frac{3}{4}x > 2$, Domain = The set of positive integers.
2. Draw the graph of the solution set of each inequality on a number line.
   a) x<6, Domain =The set of whole numbers.
   b) 2y>5, Domain = The set of integers.
   c) $\frac{1}{4}a < 2$, Domain = The set of natural numbers.
   d) y+3< -3, Domain = The set of integers.
   e) $\frac{1}{6}x < \frac{3}{4}$, Domain = The set of whole numbers.
3. Which of the following pairs of inequalities have the same solution on the set of integers?
   a) 2x>5; x>$\frac{5}{2}$
   b) $\frac{1}{4}x < 3$; x<12
   c) $\frac{3}{5}x > 2$; x<$\frac{10}{3}$
   d) x-7<1; x<8
   e) $\frac{1}{7}x - 1 > 0$; $\frac{1}{4}x > \frac{7}{4}$

4. If six times a whole number is less than 18, then find the solution and indicate the solution on a number line.

5.2 Coordinates

Activity 5.4

The grid shown has a horizontal number line and a vertical number line that meet in the middle, called the **origin.**

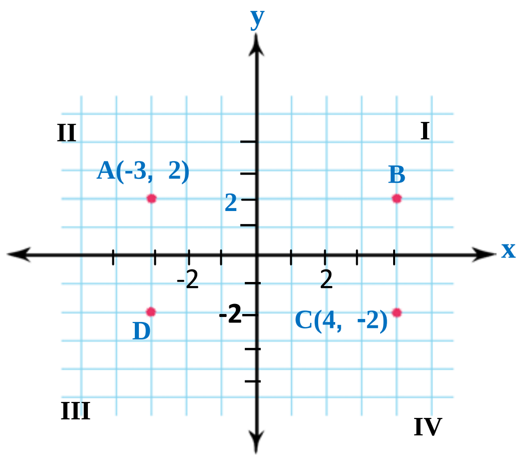


Figure 5.8

Point A is located 3 units to the left of the vertical number line and 2 units above the horizontal number line. Point A can be represented with the coordinates (-3,2).

Point C is located 4 units to the right of the vertical number line and 2 units below the horizontal number line. Point C can be represented with the coordinates (4,-2).

1. Name the coordinates that represent point B.
2. Name the coordinates that represent point D.

In mathematics, a **coordinate system**, or coordinate plane, is used to plot points in a plane. It is made up of a horizontal number line and a vertical number line that intersect at O. On the vertical number line, positive integers are represented as points above O and negative integers as points below O.

The horizontal number line is called the **x-axis**, and the vertical number line is called the **y-axis**. They intersect at their zero points. This point is called the **origin**. Together they make up a coordinate system that separates the plane into four regions called **quadrants**.

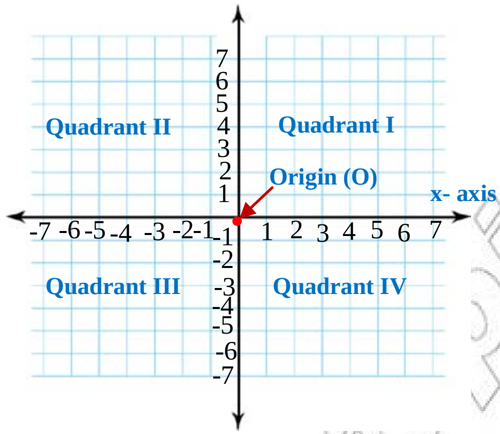


Figure 5.9

Example 10

Identify the quadrant that contains each point.

a) P

P lies in quadrant II.

b) Q

Q lies in quadrant IV.

c) R

R lies on the x-axis, between Quadrants II and III.

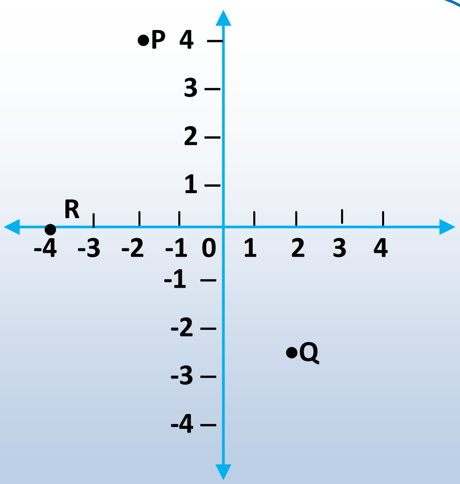


Figure 5.10

Points plotted on a coordinate system are identified by using **ordered pairs**. The first number in an ordered pair is called the **x-coordinate** (or **abscissa**), and the second number is called the **y-coordinate** (or **ordinate**).

Example 11

In the ordered pair (4,3) the x-coordinate (or abscissa) is 4 and the y-coordinate (or ordinate) is 3. What is the abscissa of the ordered pair (5,-2)? What is its ordinate?

Group work 5.2

Write down the letters at the following coordinates. What does the massage say?

(1, -4) (3, 5) (-2, 1) (4, -2)

(-5, -3) (3, 0) (3, 5) (-6, 3)

(0, -5) (1, -2) (-5, 0)

(-3, 6) (-6, 3)

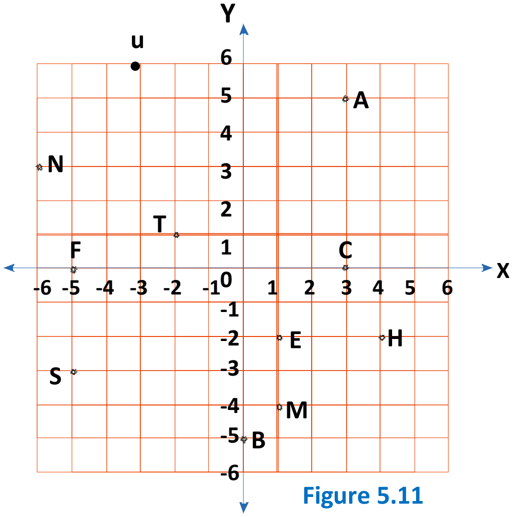


Figure 5.11

Example 12

Name the ordered pair for points A and B and identify their quadrants.

Solution: i) Start at the origin,0. Locate point A by moving right 3 units along the x-axis. The x-coordinate is +3. Now move down 5 units along the y-axis.

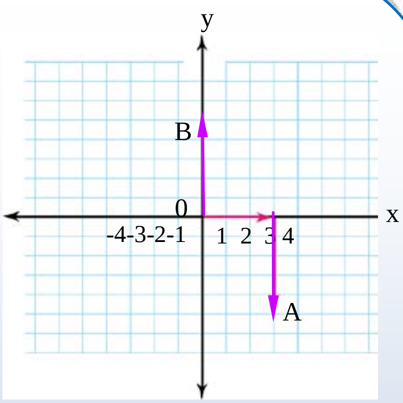


Figure 5.12

The y-coordinate is -5. The ordered pair for point A is (3,-5). Point A is in quadrant IV.

ii) We can see that point B is located on the y-axis, 4 units from the origin. The ordered pair is (0, 4). Point B is not in any quadrant because it is on an axis. What is the ordered pair for the origin?

Group work 5.3

Consider a point on the y-axis. What is the value of its abscissa? What is the value of the abscissa of any point on the y-axis?

Now consider a point on the x-axis? What is the value of its ordinate? What is the value of the ordinate of any point on the x-axis?

In order to plot a point in a coordinate system, draw a dot at the location named by its order pair. Study the following example.

Example 13

Plot the points C(3,5) and D(-3.5,0).

Solution: i) First draw a coordinate system. Start at the origin 0. Move 3 units to the right. Then move 5 units up to locate the point. Draw a dot and label it C(3,5).

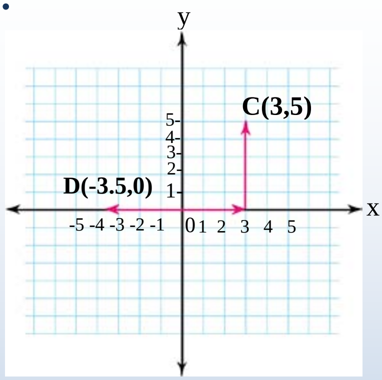


Figure 5.13

ii) To Plot the point D(-3.5,0), start at 0. Move 3.5 units to the left. Do not move up or down. Draw a dot and label it D(-3.5,0). Can you locate (-4, 2) on the coordinate plane? In which quadrant do you find it?

Exercise 5.C

1. Identify whether each of the following statements is true or false.
   a) The point p(-4, $\frac{-3}{2}$) is located in quadrant III.
   b) Q(0,-6) is a point on the x-axis.
   c) The point (3,1) satisfies the equation y=2x-5.
   d) If an ordered pair has negative x-coordinate and positive y-coordinate, then it is located in quadrant IV.
2. Write the ordered pairs corresponding to the points A,B,C,D,E and F shown in Figure 5.14.

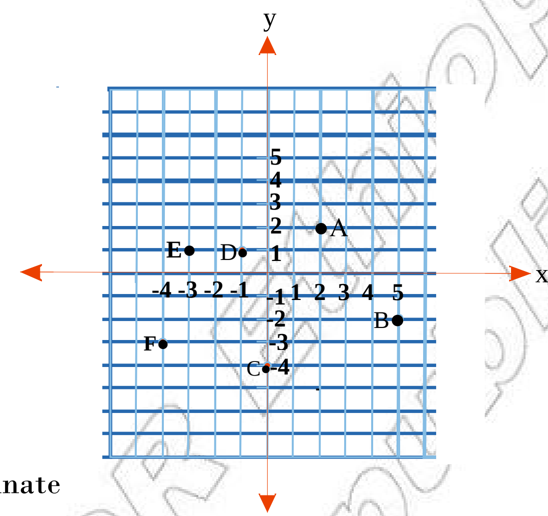


Figure 5.14

3. On graph paper, draw a coordinate plane. Then draw the graph and label each point.
   a) P(-3,6)
   b) Q(2.5,4)
   c) R(1,-4)
   d) S($\frac{-1}{2}$, -5)
   e) T(0,-3)
   f) U(4,0)
4. Name the ordered pair for each point on the city map as shown in Figure.5.15.
   a) grocery
   b) library
   c) bank
   d) city hall
   e) theater
   f) gas station

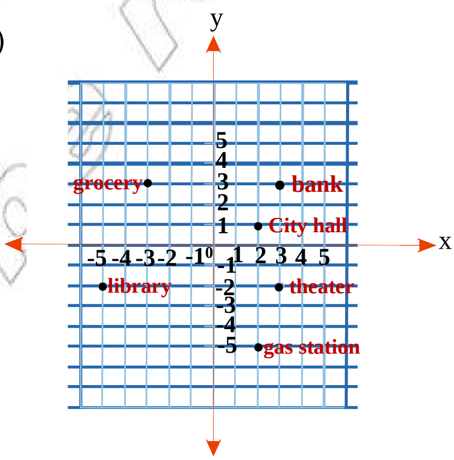


Figure.5.15

5.3. Proportionality

Activity 5.5

If y=3x, then the value of y depends on the value of x. As x varies, so does y. Simple relationships like y= 3x are customarily expressed interms of variation. How does y vary with x?

a) What will happen to y when x increases?

b) What will happen to y when x decreases?

Here you will learn the language of variation (proportionality) and how to write formulas from verbal descriptions.

5.3. 1. Direct Proportion

Suppose a car moves 60 km in one hour. The distance, d, that the car travels depends on the amount of time, t, the car takes.

Using the formula d=kt, we can write d= 60t.

Consider the possible values for t and d given in the following table.

t(hours)	1	2	3	4	5	6
d(kms)	60	120	180	240	300	360

The graph of d= 60 t is shown in figure 5.15. Note that as t gets larger, so does d.

In this situation we say that **d varies directly with t, or d is directly proportional to t.**

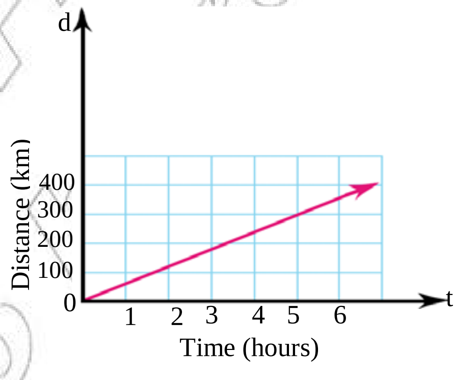


Figure 5.16

The constant rate, 60 km per hour, is called the **proportionality constant**. Join the ordered pairs (1,60), (2,120), (3,180), (4,240), (5,300) and (6,360) to get a line.

Notice that d=60 t is simply a linear equation. We are just introducing some new terms to express an old idea.

Definition 5.4: y is said to be **directly proportional** to x (written as $y \propto x$) if there is a constant k such that $y = kx$ or $\frac{y}{x} = k$.

Note: In y =kx, k is called the constant of proportionality.

Example 14

Have you observed that as the number of kilos increase the price also increases in case you go to shop to buy sugar? Suppose, in the following table, x represents number of kilos of sugar and y represents the price in Birr, what is the constant of proportionality?

x	1	2	3	4	5
y	12	24	36	48	60

Solution: Because y varies directly with x, there is a constant of proportionality k such that

y=kx

Because (for example) y=60 when x=5, we can write

60=k(5) or k=12(the constant of proportionality)

Therefore y=12x.

How much will it cost you if you buy 6 killos of sugar?

Observe that the graph of y=12x is a straight line through the origin (why?)

What do you obtain when you join the ordered pairs (1, 12), (2, 24), (3, 36), (4, 48), (5, 60)? A line?

How many kilos of sugar can you buy if you pay Birr 84?

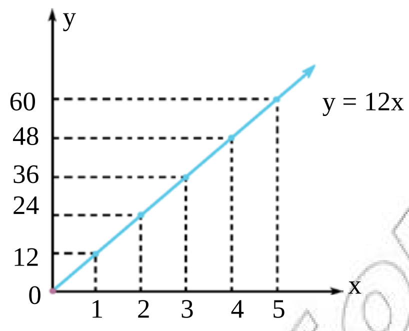


Figure 5.17

Group work 5.4

a. **Fill in the table for this rule: y = x + 3**

x	0	2	-2	-1
y			1	

b. **Fill in the gaps. Four points on the line y = x + 3 are (0, __) and (2, __), (-2, 1), (-1, __)**

c. **Plot these four points on the grid. Draw a line through them and label it y = x + 3.**

d. **Does the point (1, 4) lie on this line? Explain.**

Note. 1. **The slope of a line is a measure of its steepness.**

2. **The graph of a directly proportional relation is a straight line (where the constant of proportionality is the slope of the line) since each ordered pairs of numbers which satisfy the direct proportional relation lies on a straight line.**

Exercise 5.D

1. Find the constant of proportionality and write a formula that expresses the indicated variation.
   a) y varies directly with x, and y=12 when x=3.
   b) m varies directly with w, and $m=\frac{1}{2}$ when $w=\frac{1}{4}$.

2. Use the given formula to fill the missing entries in each table.

a) $y=\frac{3}{4}x$

y	x
$\frac{1}{3}$	
8	
	9
20	

b) $m=\frac{2}{3}w$

m	w
$\frac{1}{2}$	
3	
	6
21	

3. Solve each proportion problem.

   a) y varies directly with x, and y=100 when x=20. Find y when x=5.

   b) n varies directly with q, and n=39 when q=3. Find n when q=8.

4. A car moves at 65 km per hour. The distance traveled varies directly with the time spent traveling. Find the missing entries in the following table. Draw its graph.

Time(hours)	1	2	3	4	5
Distance(kms)					

5. The price of a cloth varies directly with the length. If a 5 metre cloth costs Birr 20, then what is the price of a 6 metre cloth?
6. If Ayele can bicycle 25 km in 2 hours, then how far can he bicycle in 5 hours?
7. If Mamitu can extract 3 kg of butter from 21 litres of milk, then how much butter can she extract from 84 litres of milk?

5.3.2. Inverse Proportion

Activity 5.6

If yx = 10, how does y vary with x?

a) What will happen to y when x increases?

b) What will happen to y when x decreases?

If you plan to make a 400 km trip by car, the time it will take depends on your rate of speed. Using the formula d=vt, we can write $t=\frac{400}{v}$

Consider the possible values for v and t given in the following table:

v(km/hr)	10	20	40	50	80	100
t(hours)	40	20	10	8	5	4

The graph of $t=\frac{400}{V}$ is shown in Figure5.17. You may join the ordered pairs (10,40), (20,20), (40,10), (50,8), (80,5), (100,4) to get such a curve. As your rate increases, the time for the trip decreases. In this situation we say that the time is **inversely proportional** to speed. Note that the graph of $t=\frac{400}{V}$ is not a straight line because $t=\frac{400}{V}$ is not a linear equation.

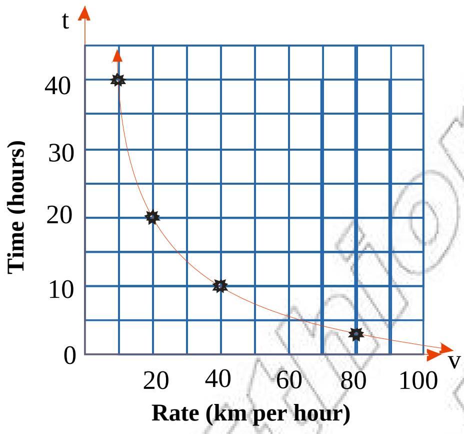


Figure 5.18

Definition 5.5: y is said to be inversely (indirectly) proportional (written $y\propto\frac{1}{x}$) if there is a constant k such that $y=\frac{k}{x}$ or y.x=k.

Example 15

The following situations are some examples of inverse proportional relations.

a) **The time taken to finish a piece of work and the number of people doing the work.**
b) **The volume of a gas at a constant temperature and pressure.**
c) **The steady speed of a car and the time it takes to cover a fixed distance.**

Example 16

A prize of Birr 80,000 is to be shared equally among x winners of a game.

If y represents the share of each winner, then y=$\frac{80,000}{x}$.

Answer the following.

a) **If 40 people share the prize, what is the share of each?**

b) **If the share of each winner is Birr 400, then how many people won the game?**

c) **Find the constant of proportionality?**

Solution: **a) x=40, Y=** $\frac{80,000}{40} = 2,000$

Therefore, the share of each people will be Birr 2,000.

b) Y= 400, then 400= $\frac{80,000}{x}$**. Thus 400x=80,000 (why?)**

or x= $\frac{80,000}{400}$ **=200**

therefore, 200 people won the prize.

c) Y=$\frac{k}{x}$ **implies k=80,000**

Therefore, the constant of proportionality equals 80,000.

Exercise 5.E

1. Find the constant of proportionality, and write a formula that expresses the indicated variation.
   a) y varies inversely with x, and y=3 when x=2.
   b) c varies inversely with d, and c=5 when d=2.
   c) a varies directly with b, and a=3 when b=4.

2. Solve each variation problem.
   a) y varies directly with x, and y=100 when x=20. Find y when x=5.
   b) a varies inversely with b, and a=9 when b=2.
      Find a when b=6.
3. Use the given formula to fill the missing entries in each table and determine whether b varies directly or inversely with a.

   i) $b=\frac{300}{a}$ ii) $b=\frac{500}{a}$ iii) $b=\frac{3}{2}a$

a	b
$\frac{1}{2}$	
1	
	10
900	

a	b
4	
	$\frac{1}{2}$
250	
$\frac{1}{8}$	

a	b
12	
	24
	$\frac{9}{4}$
15	

4. For each table, determine whether y varies directly or inversely with x and find a formula for y interms of x.

   a)

x	y
10	5
15	7.5
20	10
25	12.5

   b)

x	y
2	10
4	5
10	2
20	1

   c)

x	y
2	7
3	10.5
4	14
5	17.5

5. The time that it takes to complete a 300 km trip varies inversely with your average speed. Fill in the missing entries in the following table.

speed (km/hr)	20	40	50	
time (hours)				2

6. A bag contains sweets. When divided among 20 children each child receives 8 sweets. If the sweets were divided among 32 children, how many would each receive?
7. A train traveling at a speed of 80 km/hr will take 9 hours to cover the distance between two cities. How long will it take a car traveling at 60 km/hr to cover the same distance?

UNIT SUMMARY

Important facts you should know:

- A mathematical statement of equality which involves one or more variables is called an **equation**.
- An equation that can be written in the form $ax+b=0$, $a \neq 0$ is called **a linear equation**.
- The set whose elements are considered as possible replacement for the variable in a given equation or inequality is called the **domain of the variable**.
- While solving an equation, the following operations may be carried out without changing the equation:
  1. Add the same number to both sides of the equation.
  2. Subtract the same number from both sides of the equation.
  3. Multiply both sides of the equation by the same non-zero number and
  4. Divide both sides of the equation by the same non-zero number.
- While solving inequalities, use the following rules of transformation.
  1) Adding or subtracting the same number to or from each side of an inequality keeps the inequality sign remain as it is.
  2) Multiplying or dividing both sides of an inequality by the same positive number keeps the inequality sign as it is.
- **A coordinate system, or coordinate plane** is used to plot points in a plane. It is made up of a horizontal number line and a vertical number line that intersect at the origin.
- y is said to be **directly proportional** to x if there is a constant k such that $y=kx$. K is called the constant of proportionality.
- y is said to be **inversely (indirectly) proportional** to x if there is a constant k such that $y = \frac{k}{x}$

Review Exercise

1. Which expression has a value of 74 when $x = 10$, $y = 8$ and $z = 12$?
   a. $4xyz$ c. $x + 5y + 2z$
   b. $2xz - 3y$ d. $6xyz + 8$
2. Which expression simplifies to $9x + 3$ when you combine like terms?
   a. $10x^2 - x^2 - 3$ c. $3x + 7 - 4 + 3x$
   b. $18 + 4x - 15 + 5x$ d. $7x^2 + 2x + 6 - 4$
3. What is the solution of the equation $810 = x - 625$?
   a. $x = 185$ c. $x = 725$
   b. $x = 845$ d. $x = 1,435$
4. Solve each of the following linear equations
   a. $x - \frac{1}{4} = \frac{3}{5}$ c. $2x = \frac{1}{3}$
   b. $x + \frac{1}{5} = 2$ d. $\frac{3}{4}x = 81$
5. Solve each of the following linear inequalities on the given domain.
   a. $x - \frac{1}{4} < \frac{1}{5}$, domain = the set of whole numbers.
   b. $x + \frac{2}{3} > 4$, domain= the set of counting numbers.
   c. $3x < \frac{3}{7}$, domain = the set of negative integers.
   d. $\frac{1}{2}x > \frac{3}{5}$, domain = the set of negative integers.
6. The perimeter of a square is four times the length of one of its sides. What is the perimeter of a square whose side has length 21 cm?

7. Name the point with the given coordinates (Figure 5.19).
   a) (8, 0)________
   b) (1, 4)________
   c) (-5, 6) _______
   d) (6, -5)________
   e) (8, -8)________
   f) (-7, -4)________
   g) (-2, 0)________
   h) (-5, -3)_______
   i) (-8, 0) _______
   j) (0, -8) _______

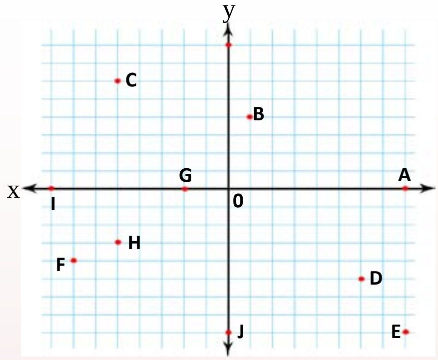


Figure 5.19

8. Without graphing, tell whether (in Figure 5.19) the line containing each pair of points is vertical or horizontal.
   a) I and G b) H and C c) A and E

9. When a weight is hung on a spring, the extension produced on the spring is directly proportional to the weight.
   a) Find x(Figure 5.20)
   b) Find the amount that the spring will stretch with a weight of 6 kg (Fig.5.21)

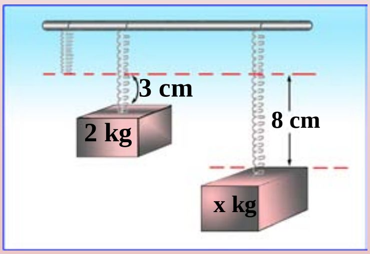


Figure 5.20

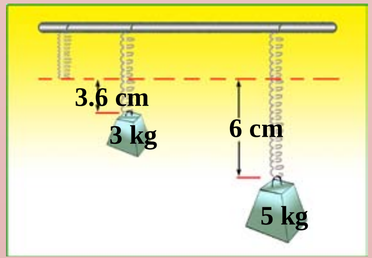
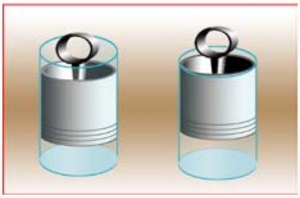


Figure 5.21

10. The volume of a gas in a cylinder is inversely proportional to the pressure on the gas. If the volume is 12 cubic centimeters when the pressure on the gas is 200 kilograms per square centimeter, then what is the volume when the pressure is 150 kilograms per square centimeter? (figure 5.22)



$V = 12\ cm^3$ $\quad V = ?$

$P = 200 \frac{kg}{cm^2}$ $\quad P = 150 \frac{kg}{cm^2}$

Figure 5.22

11. Identify whether the given relation is directly proportional or inversely proportional.

a) The number of children in a family and the share of their father's fortune.

b) The speed of a car and the time it takes to cover a fixed distance.

c) The length of a rectangle of constant area with the width.

12. Wallpaper for a bedroom costs Birr 16 per roll for the walls and Birr 9 per roll for the border. If the room requires 12 rolls of paper for the walls and 6 rolls for the border, find the total cost for decorating the bedroom.